Math 12Honors Ch6: COMPLEX Number ReVIEW

Question 1 (**)

$$w=\frac{-9+3\mathrm{i}}{1-2\mathrm{i}}.$$

Find the modulus and the argument of the complex number w.

Question 2 (**)

Solve the equation

$$2z^2-2\mathrm{i}z-5=0,\ z\in\mathbb{C}.$$

Question 3 (**)

Find the value of x and the value of y in the following equation, given further that $x\in\mathbb{R}$, $y\in\mathbb{R}$.

$$(x+\mathrm{i}y)(2+\mathrm{i})=3-\mathrm{i}.$$

Question 4 (**)

$$z=\frac{\lambda+4\mathrm{i}}{1+\lambda\mathrm{i}},\ \lambda\in\mathbb{R}.$$

Given that z is a real number, find the possible values of λ.

Question 5 (**)

Find the values of x and y in the equation

$$x(1+\mathrm{i})^2+y(2-\mathrm{i})^2=3+10\mathrm{i},\ x\in\mathbb{R},\ y\in\mathbb{R}.$$

Question 6 (**)

Find the value of x and the value of y in the following equation, given further that $x\in\mathbb{R}$, $y\in\mathbb{R}$.

$$(x+\mathrm{i}y)(3+4\mathrm{i})=3-4\mathrm{i}.$$

Question 7 (**)

The complex number z satisfies the equation

$$4z-3\overline{z}=\frac{1-18\mathrm{i}}{2-\mathrm{i}},$$

where $\overline{z}$ denotes the complex conjugate of z.

Solve the equation, giving the answer in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 8 (**)

$$z=-3+4\mathrm{i} \quad \text{and} \quad zw=-14+2\mathrm{i}.$$

By showing clear workings, find …

a) … w in the form $a+b\mathrm{i}$, where a and b are real numbers.

b) … the modulus and the argument of w.

Question 9 (**)

$$z=22+4\mathrm{i} \quad \text{and} \quad \frac{z}{w}=6-8\mathrm{i}.$$

By showing clear workings, find …

a) … w in the form $a+b\mathrm{i}$, where a and b are real numbers .

b) … the modulus and the argument of w.

Question 10 (**)

$$z=(2-\mathrm{i})^2+\frac{7-4\mathrm{i}}{2+\mathrm{i}}-8.$$

Express z in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 11 (**)

The complex conjugate of z is denoted by $\overline{z}$.

Solve the equation

$$2z-3\overline{z}=\frac{-27+23\mathrm{i}}{1+\mathrm{i}},$$

giving the answer in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 12 (**+)

Solve the following equation.

$$z^2=21-20\mathrm{i}, \quad z\in\mathbb{C}.$$

Give the answers in the form $a+b\mathrm{i}$, where $a\in\mathbb{R}$ and $b\in\mathbb{R}$.

Question 13 (**+)

The cubic equation

$$2z^3-5z^2+cz-5=0, \quad c\in\mathbb{R},$$

has a solution $z=1-2\mathrm{i}$.

Find in any order …

a) … the other two solutions of the equations.

b) … the value of c.

Question 14 (**+)

The quadratic equation

$$z^2-2z+1-2\mathrm{i}=0,\ c\in\mathbb{R},$$

has a solution $z=-\mathrm{i}$.

Find the other solution.

Question 15 (**+)

$$z-8=\mathrm{i}(7-2\overline{z}),\ z\in\mathbb{C}.$$

The complex conjugate of z is denoted by $\overline{z}$.

Determine the value of z in the above equation, giving the answer in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 16 (**+)

$$z^3+Az^2+Bz+26=0,\text{ where } A\in\mathbb{R},\ B\in\mathbb{R}$$

One of the roots of the above cubic equation is $1+\mathrm{i}$.

a) Find the real root of the equation.

b) Determine the values of A and B.

Question 17 (**+)

The complex conjugate of z is denoted by $\overline{z}$.

Solve the equation

$$z-12=\mathrm{i}(9-2\overline{z}),$$

giving the answer in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 18 (**+)

The complex number z satisfies the equation

$$2z-\mathrm{i}\overline{z}=3(3-5\mathrm{i}),$$

where $\overline{z}$ denotes the complex conjugate of z.

Determine the value of z, giving the answer in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 19 (**+)

The cubic equation

$$2z^3-z^2+4z+p=0,\ p\in\mathbb{R},$$

is satisfied by $z=1+2\mathrm{i}$.

a) Find the other two roots of the equation.

b) Determine the value of p.

Question 20 (**+)

Solve the following equation.

$$w^2=5-12\mathrm{i},\quad w\in\mathbb{C}.$$

Give the answers in the form $a+b\mathrm{i}$, where $a\in\mathbb{R}$ and $b\in\mathbb{R}$.

Question 21 (**+)

$$z = 1 + \sqrt{3}\mathrm{i} \quad \text{and} \quad \frac{w}{z} = 2 + 2\mathrm{i}\,.$$

Find the exact value of the modulus of w and the exact value of the argument of w.

Question 22 (**+)

The following cubic equation is given

$$z^3 + az^2 + bz - 5 = 0\,,$$

where $a \in \mathbb{R}$, $b \in \mathbb{R}$.

One of the roots of the above cubic equation is $2 + \mathrm{i}$.

a) Find the other two roots.

b) Determine the value of a and the value of b.

Question 23 (**+)

The following cubic equation is given

$$z^3 + pz^2 + 6z + q = 0\,,$$

where $p \in \mathbb{R}$, $q \in \mathbb{R}$.

One of the three solutions of the above cubic equation is $5 - \mathrm{i}$.

a) Find the other two solutions of the equation.

b) Determine the value of p and the value of q.

Question 24 (**+)

The complex number z is defined as

$$z = \mathrm{i}(1+\mathrm{i})(1-2\mathrm{i})^2\,.$$

It is further given that

$$\overline{z - 3\mathrm{i}} + P(z - 3\mathrm{i}) = Q\,\overline{z}$$

where P and Q are **real** constants.

Find the value of P and the value of Q.

Question 25 (***)

$$z = \sqrt{3} + \mathrm{i} \quad \text{and} \quad w = 3\mathrm{i}\,.$$

a) Find, in exact form where appropriate, the modulus and argument of z and the modulus and argument of w.

b) Determine simplified expressions for zw and $\frac{w}{z}$, giving the answers in the form $x + \mathrm{i}y$, where $x \in \mathbb{R}$, $y \in \mathbb{R}$.

c) Find, in exact form where appropriate, the modulus and argument of zw and the modulus and argument of $\frac{w}{z}$.

Question 26 (***)

Find the value of x and the value of y in the following equation, given further that $x\in\mathbb{R}$, $y\in\mathbb{R}$.

$$\frac{1}{x+\mathrm{i}y}-\frac{1}{1+\mathrm{i}}=2-3\mathrm{i}\,.$$

Question 27 (***)

Find the square roots of $1+\mathrm{i}\sqrt{3}$.

Give the answers in the form $a+b\mathrm{i}$, where $a\in\mathbb{R}$ and $b\in\mathbb{R}$.

Question 28 (***)

Solve the equation

$$\frac{13z}{z+1}=11-3\mathrm{i}\,,\quad z\in\mathbb{C}\,,$$

giving the answer in the form $x+\mathrm{i}y$, where x and y are real numbers.

Question 29 (***)

The complex conjugate of w is denoted by $\overline{w}$.

Given further that

$$w=1+2\mathrm{i}\ \text{ and }\ z=w-\frac{25\overline{w}}{w^2}\,,$$

show clearly that z is a real number, stating its value.

Question 30 (***)

The following cubic equation is given

$$z^3+2z^2+az+b=0\,,$$

where $a\in\mathbb{R}$, $b\in\mathbb{R}$.

One of the roots of the above cubic equation is $1+\mathrm{i}$.

a) Find the real root of the equation.

b) Find the value of a and the value of b.

Question 31 (***)

The following complex numbers are given.

$$z_1=2-2\mathrm{i}\,,\quad z_2=\sqrt{3}+\mathrm{i}\ \text{ and }\ z_3=a+b\mathrm{i}\ \text{ where }\ a\in\mathbb{R}\,,\ b\in\mathbb{R}\,.$$

a) If $|z_1z_3|=16$, find the modulus z_3.

b) Given further that $\arg\left(\frac{z_3}{z_2}\right)=\frac{7\pi}{12}$, determine the argument of z_3.

c) Find the values of a and b, and hence show $\frac{z_3}{z_1}=-2$.

Question 32 (***)

Solve the equation

$$2z^4-14z^3+33z^2-26z+10=0\,,\quad z\in\mathbb{C}$$

given that one of its roots is $3+\mathrm{i}$.

Q1 $\lvert w\rvert = 3\sqrt{2}$, $\arg w = -\frac{3\pi}{4}$	Q7 $z = 4 - \mathrm{i}$	Q13 $z_2 = 1 + 2\mathrm{i}$, $z_3 = \frac{1}{2}$, $c = 12$	Q19 $1 - 2\mathrm{i}, -\frac{3}{2}$, $p = 15$
Q2 $z = \pm\frac{3}{2} + \frac{1}{2}\mathrm{i}$	Q8 $w = 2 + 2\mathrm{i}$, $\lvert w\rvert = 2\sqrt{2}$, $\arg w = \frac{\pi}{4}$	Q14 $z_2 = 2 + \mathrm{i}$	Q20 $w = \pm(3 - 2\mathrm{i})$
Q3 $(x, y) = (1, -1)$	Q9 $w = 1 + 2\mathrm{i}$, $\lvert w\rvert = \sqrt{5}$, $\arg w \approx 1.11^{\mathrm{c}}$	Q15 $z = 2 + 3\mathrm{i}$	Q21 $\lvert w\rvert = 4\sqrt{2}$, $\arg w = \frac{7\pi}{12}$
Q4 $\lambda = \pm 2$	Q10 $-3 - 7\mathrm{i}$	Q16 $z = -13$, $A = 11$, $B = -24$	Q22 $z_2 = 2 - \mathrm{i}$, $z_3 = 1$, $a = -5$, $b = 9$
Q5 $x = 7$, $y = 1$	Q11 $z = 2 + 5\mathrm{i}$	Q17 $z = 2 + 5\mathrm{i}$	Q23 $z_2 = 5 + \mathrm{i}$, $z_3 = 2$, $p = -8$, $q = 52$
Q6 $(x, y) = \left(-\frac{7}{25}, -\frac{24}{25}\right)$	Q12 $z = \pm(5 - 2\mathrm{i})$	Q18 $z = 1 - 7\mathrm{i}$	Q24 $P = 3$, $Q = 4$

Q25 $\lvert z\rvert = 2$, $\lvert w\rvert = 3$, $\arg z = \frac{\pi}{6}$, $\arg w = \frac{\pi}{2}$, $zw = -3 + 3\sqrt{3}\mathrm{i}$, $\frac{w}{z} = \frac{3}{4} + \frac{3}{4}\sqrt{3}\mathrm{i}$, $\lvert zw\rvert = 6$, $\left\lvert\frac{w}{z}\right\rvert = \frac{3}{2}$, $\arg(zw) = \frac{2\pi}{3}$, $\arg\left(\frac{w}{z}\right) = \frac{\pi}{3}$	Q31 $\lvert z_3\rvert = 4\sqrt{2}$, $\arg z_3 = \frac{3\pi}{4}$, $a = -4$, $b = 4$
Q26 $(x, y) = \left(\frac{5}{37}, \frac{7}{37}\right)$	Q32 $z = 3 + \mathrm{i}$, $z = 3 - \mathrm{i}$, $z = \frac{1}{2} + \frac{1}{2}\mathrm{i}$, $z = \frac{1}{2} - \frac{1}{2}\mathrm{i}$
Q27 $\pm\frac{1}{2}\left(\sqrt{6} + \mathrm{i}\sqrt{2}\right)$	
Q28 $z = 1 - 3\mathrm{i}$	
Q29 12	
Q30 $z = -4$, $a = -6$, $b = 8$	